Application of Cooperative Diversity to Cognitive Radio Leasing: Model and Analytical Characterization of Resource Gains

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***Abstract*—This paper studies mutual information in a novel scenario combining cooperative diversity and cognitive radio based on spectrum leasing. In the scenario there is a primary transmitter subject to random channel fading, with full spectrum rights available, and a secondary transmitter without any spectrum rights, which offers its cooperation to the primary transmitter in exchange for a share of the resulting resource gains. Two decision-making schemes, based on different levels of channel knowledge, are considered in the primary transmitter. These are knowledge of the statistics of random fading (averages) and full knowledge of instantaneous channel state. The contributions of this paper are the radio leasing scenario itself, which, unlike previous approaches in cognitive radio, is not based on game theory, and the statistical characterization of the resource gains achieved in this scenario using cooperative diversity for spectrum leasing under diverse channel conditions and the aforementioned decision-making schemes. Analytical expressions are obtained for the probability of cooperation, the mutual information probability density function and its average, and the proportion of resource gains achieved for the statistical and instantaneous channel knowledge schemes. We identify the conditions for resource gains: for statistical channel knowledge to suffice, the primary link must be of low quality, whereas for instantaneous channel knowledge, although resource gains are achieved in any situation, they increase as the primary channel gets worse.**

***Index Terms*—Wireless networks, cooperative diversity, cognitive radio.**

I. INTRODUCTION

THIS paper provides a theoretical analysis of a novel cognitive cooperative diversity scenario with applications in cognitive radio. Unlike previous cooperative diversity models, our model has a primary transmitter with full spectrum rights and a secondary transmitter, willing to gain access, which offers its help to increase the mutual information (M.I.) of the primary transmitter in exchange for a portion of the spectrum resource *gains*. By resource gains we refer to the extra capacity that becomes available thanks to the improved efficiency achieved by cooperation. The main contributions of the paper are, thus, two: Firstly, we present a new model and secondly, we present a statistical characterization of resource gains that result from the application of cooperative diversity to spectrum leasing under varying channel conditions and different decision-making scenarios.

We determine the conditions in which the secondary transmitter helps to increase M.I. and provide results for two scenarios. The first assumes partial knowledge of the statistics of random fading (statistical channel state information [CSI]) and the second assumes perfect knowledge of the channel (instantaneous CSI). Our analysis shows that M.I. increases in these decision-making scenarios in particular circumstances. In the first scenario, the average M.I. will only improve if the primary channel is suffering an important degradation. In the second scenario, M.I. always increases, with a probability that grows with the degradation of the primary channel.

A common technique to combat fading (random variations of channel attenuation) is diversity, which consists of transmitting several replicas of a signal. When the replicas are separated in time, this diversity is called time-diversity. Similarly, frequency-diversity and space-diversity result from the utilization of different frequencies or spaced antennas for the same signal [1]. Space-diversity is appealing because it can complement other forms of diversity. However, it could not be exploited by single-antenna devices until the advent of cooperative diversity. In cooperative diversity, single-antenna networked nodes cooperate to form a virtual antenna array that supports space-diversity [1], [2].

Typically, cooperative diversity transmission involves two phases. First, the source node attempts to transmit information towards the destination while neighbor nodes simultaneously store the information they overhear. In the second phase, this stored information is relayed by one (or several) of those nodes.

To the best of our knowledge, all previous studies of M.I. in cooperative diversity systems assume that the channel is evenly shared, which in practice may mean that all nodes feature a fair MAC protocol [2], [3], [4], [5]. In order to apply cooperative diversity to cognitive radio, we are interested in

Manuscript received May 5, 2011; revised April 11, 2012; accepted October 4, 2012. The associate editor coordinating the review of this paper and approving it for publication was T. J. Li.

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This research has been supported by projects MEFISTO (10TIC006CT), funded by Xunta de Galicia, Spain, and CALM (TEC2010-21405-C02-01), funded by MICINN, Spain.

Digital Object Identifier 10.1109/TWC.2012.113012.110842

analyzing M.I. without this assumption. In our model, one user has full spectrum rights while a second one must provide a collaboration payment in order to gain access. The goal is to obtain net resource gains relative to a model without collaboration. Although the way in which gains are shared is not the focus of our study, we suggest that they should be divided evenly between the two transmitters.

Spectrum leasing is a spectrum reuse technique where a primary transmitter leases part of its resources to a secondary transmitter [6]. We assign all decision-making to the primary transmitter, which owns the spectrum. The transmitter is responsible for switching its transmission mode from direct mode to collaboration mode when the latter produces net resource gains, and for making part of these resources available to collaborators who are notified. In the most common cognitive radio approaches, however, there is no collaboration. Secondary nodes simply transmit in the white spaces of spectrum that belongs to the primary transmitter, independently of its transmission being efficient or not. The instantaneous CSI knowledge scenario is only feasible in practice if the resource gains are worth the overhead. The analysis of this case, which assumes a genie is available, shows an upper bound on achievable resource gains in practical scenarios. When the channel varies too rapidly to be tracked, only statistical (average) CSI can be used.

The rest of this paper is organized as follows. In section II we discuss related work. In section III we describe our model. In section IV we characterize the probability of increasing M.I. by cooperation. In section V we analyze M.I. as a random variable. In section VI we obtain the closed expressions for the average M.I. and apply them to decisions based on statistical CSI. In section VII we analyze the instantaneous CSI knowledge scenario. Finally, section VIII concludes the paper.

II. Related Work

Goldsmith et al. reviewed cognitive radio in [7]. They described diverse information theoretical solutions to the problem of a secondary transmitter gaining access to the spectrum without affecting the spectrum rights of the owner, and identified different forms of cooperation between cognitive and non-cognitive users. According to their definitions [7], our model belongs to the category of "aware non-cognitive users".

Although the concept of *spectrum leasing* is known in cognitive radio, we follow an approach based on cooperative diversity that we characterize analytically with information theory. Cooperative leasing schemes were studied in [6] and [8], which proposed negotiating resource gains leading to performance improvement as an alternative to money transactions for spectrum leasing. However, in the model described in [6], each node makes decisions on its own transmissions, which means that the secondary transmitter can improve its information flow by reducing the power it allocates to collaboration. In the Nash equilibrium of the game theoretic solution, the authors showed that there is a global maximum. Our model, in contrast, assumes that the primary transmitter, with full spectrum rights, directly chooses the proportion of resources to be allocated to each information flow (see Fig. 1), thereby controlling the fraction of resources that the secondary transmitter receives for transmission (Fig. 2). This is similar to, yet more efficient than, traditional white-space-oriented cognitive radio in which secondary transmitters are only allowed to use free primary spectrum.

With our model, we seek to improve the distribution of M.I. Note that other alternatives are possible. For example, in [9] the authors designed a spectrum leasing system that simply guarantees that the outage probability of the primary transmitter will not increase.

The fractional cooperation schema in [10] may seem similar to ours, but the difference is significant. In fractional cooperation, the secondary transmitter is allowed to relay just the minimum information necessary to achieve full diversity [2]. The remaining resources are employed for secondary transmission. In other words, fractional cooperation obtains resources by keeping cooperation to a minimum, whereas our model does not restrict cooperation as it extracts resource gains from the increase in M.I. at the primary transmitter.

Regarding the implementation of the leasing mechanism, in [11] the authors present a cognitive spectrum leasing system with cooperation from the perspective of packet queuing and retransmission. In their system, the primary transmitter accepts packet acknowledgements from either the destination or the secondary transmitters, which can deliver primary packets. A throughput analysis of the different mechanisms is proposed. This implementation can be considered for any of the aforementioned models, including ours.

Finally, previous characterizations of *decode-and-forward* (DF) relaying gains can be found in [12]. They characterize the probability that a two-hop DF relay channel requires a lower *signal-to-noise ratio* (SNR) than that of a direct transmission. We apply similar formulations, but our analysis is more extensive since we quantify the proportion of resource gains, and we also consider cooperative DF [2].

III. System Model

We consider a wireless link between a primary source S_P and a primary destination D_P, with an SNR without fading of $\gamma = E_s/N_0$ through a channel with random Gaussian fading with coefficient $a_{sd} \sim \mathcal{N}(0, \sigma_{sd}^2)$. The transmission rate is originally limited by the M.I. of the direct channel [2]:

$$I_D = \log_2(1 + |a_{sd}|^2\gamma) \tag{1}$$

Our goal is to study the effect of introducing a secondary transmitter S_S that wants extra resources to transmit its own information to its own destination. The result is the interference channel shown in figure 1. The sequence of actions taken by S_S is the following: Firstly, if S_S has its own channel, it will use it (this is not reflected in our model); secondly, it will try to find white spaces, thereby avoiding the need to cooperate and waste energy in this process (this is not reflected in our model either); finally, if it still needs resources, or if none of the previous alternatives are available, it will offer cooperation to inefficient primaries according to our model. In such a case, cooperation may yield resource gains which the primary transmitter will share with S_S.

Like in [2], we assume that wireless nodes are half duplex and that transmissions are not overlapped by assigning a

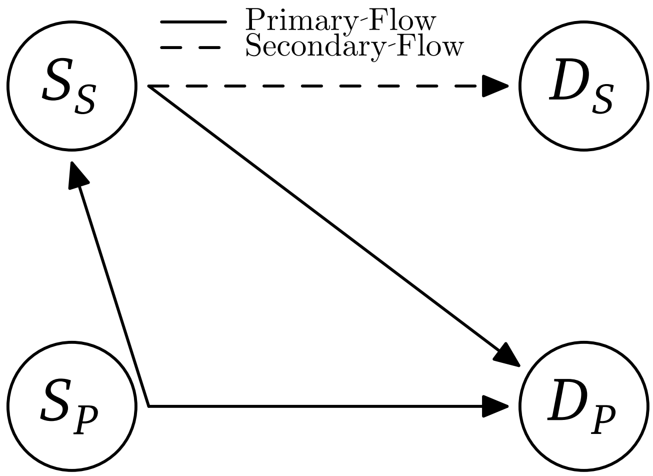

Figure 1. Wireless network with four nodes. If S_S is not present, the primary transmission from S_P to D_P occupies the entire medium. S_S can overhear the primary transmission and in some cases enhance it by relaying information. In such a case, resource gains may be shared with a secondary information flow from S_S to D_S, as a reward for cooperation. S_P, the spectrum owner, controls the whole process.

channel slot to a single transmitter at a time. This places fewer homogeneity constraints, and allows cooperation between technologies that do not support overlapping. Our analytical results are obtained using the cooperative DF protocol [2] but more powerful relaying techniques would achieve better gains.

Two levels of channel knowledge are considered: knowledge of average fading (statistical CSI) and full knowledge of instantaneous CSI. The former is more adequate for channels that change faster than the response time of the measurement technique. The latter is more suitable for a slower channel variation, that may appear in some realistic cases. Obviously, a heavy overhead of channel measurement may degrade performance. Since we have deliberately ignored this possibility and assumed that instantaneous CSI is available, our results set an upper bound on the benefits achievable in any practical system. In other words, in the second case we assume that a genie informs the decisor of the exact channel state. This means that the M.I. gain analysis is independent of the MAC protocol.

The statistical (D_s) and instantaneous CSI criteria (D_i) can be expressed as follows:

$$D_s = \arg\max(E[I_D], E[I_C]) \tag{2}$$

$$D_i = \arg\max(I_D, I_C) \tag{3}$$

where I_C is the M.I. of cooperative transmission. Node S_P takes all the decisions, as the owner of the spectrum. S_P must reconfigure its transmission mechanism and inform S_S of its decision. There are many proposals in cooperative diversity for mechanisms that allow channel measurement and signaling [13]. If channel variations are slow, one possibility would be to grant the nodes access to a shared database [14]. Faster varying channels can be estimated from control packets as in the three-way handshake mechanism described in [15]. If channel variation is too fast for any of the mechanisms available, the statistical CSI approach is the only practical alternative. Regarding the physical layer, two DF receivers are described in [15]: separate decode attempts for each version of the packet received and the application of Maximum Rate Combining (MRC) to exploit the information in the two signals received.

When the primary transmitter and the secondary follow different radio standards, exploiting cooperative diversity imposes certain compatibility requirements, as the secondary radio must be able to receive and retransmit S_P signals. However, this may not be a limiting assumption, since new paradigms such as *software defined radio* (SDR) suggest that hardware reconfigurability will be easily achievable in future devices.

To illustrate the relation with other approaches in cooperative diversity, let us compare our approach with a fair MAC protocol that arbitrates the accesses of S_P and S_S to the medium.

A. Two nodes with equal rights: fair MAC protocol

If a fair MAC is used by S_P and S_S, the two nodes will receive an equal share of resources. The usual approach is the well-known MAC channel, but, as previously mentioned, we decided to follow a simpler architecture, such as that described in [2], where equal proportions of channel resources are granted to each user and overlapping transmissions are not allowed. Thus the model is reduced to two parallel single-user channels and the rate of each node i using protocol x is limited by:

$$R_i \leq \frac{I_{x,i}}{2} \tag{4}$$

where the division by 2 represents a fair medium sharing and $I_{x,i}$ is the achievable M.I. for node i under protocol x. x may refer to direct transmission or cooperative transmission. For example, for cooperative diversity with one DF relay, the M.I. of node i combining direct and relayed receptions is given by [2] as:

$$I_{DF,i} = \frac{1}{2}\log_2(1 + \gamma \min(|a_{sr}|^2, |a_{sd}|^2 + |a_{rd}|^2)) \tag{5}$$

where a_{sr} and a_{rd} are the fading coefficients of the source-relay and relay-destination channels, respectively.

B. Two nodes with different rights: traditional cognitive radio and the new model

If nodes S_P and S_S do not use a fair MAC protocol, resource assignment may be asymmetric. In many commercial communications, a licensed operator can retain empty frequency bands. This may lead to a considerable waste of resources if operator transmission is inefficient.

In a typical cognitive radio model, S_S would use its knowledge of the primary transmission to access free portions of the spectrum (white spaces) or to transmit its signal overlapping the primary transmission spectrum using techniques that prevent degradation of primary reception. Thus, traditional cognitive radio assumes that S_P does not share the spectrum it uses. Our approach, in contrast, assumes that S_P can lease out spectrum resources that are not utilized efficiently in exchange for cooperation from S_S. This leads to net resource gains, which S_P, as the spectrum owner, controls.

To characterize resource sharing, let $\alpha \in [0, 1]$ be a scalar representing the proportion of channel that S_P releases to S_S.

Figure 2. Time division schema for resource leasing characterized by α. S_P has certain knowledge of the channel and decides the value of α. In the first stage S_P transmits towards D_P and S_S overhears the signal. In the second stage S_S relays the primary information. Finally, S_S is granted a fraction αT of resource gains for it own secondary transmission.

For example, in a TDMA schema such as that shown in figure 2, αT would represent the interval for secondary information transmission and $(1-\alpha)T$ the interval for primary information transmission, which includes cooperative (assisted) transmission. In OFDMA, α would be the proportion of carriers that S_S would borrow for the transmission of secondary information [8]. The maximum fraction of resources that the primary transmitter could release without rate degradation, α_{th}, is defined as follows:

$$I_D \leq (1-\alpha)I_C \Rightarrow \alpha_{th} = (1-\frac{I_D}{I_C}) \tag{6}$$

Note that (6) may yield negative values for α_{th}.Thus, for the case with instantaneous CSI knowledge, we define the maximum fraction of resource gains for each channel state as:

$$\alpha_{MAX} = \max(0, \alpha_{th}) \tag{7}$$

For decisions based on statistical CSI, the fraction of resources gained is lower because α tracks channel variations coarsely. The maximum proportion $\tilde{\alpha}_{MAX}$ of releasable resources is similar to (7), but averages are used as indicated in (2):

$$\tilde{\alpha}_{MAX} = \max\left(0, 1-\frac{E\left[I_D\right]}{E\left[I_{DF}\right]}\right) \tag{8}$$

Expressions (7) and (8) determine cooperation profitability. They are zero when cooperation does not produce resource gains. In other words, when $\alpha_{MAX} > 0$, there is a potential gain that can be exploited by selecting values $\alpha \in [0, \alpha_{MAX}]$ for instantaneous CSI knowledge, and values $\alpha \in [0, \tilde{\alpha}_{MAX}]$ for statistical CSI knowledge.

We can suggest different policies depending on the needs of the primary transmitters. Next, we list some policies for the full CSI knowledge scenario, but α_{MAX} can be replaced with $\tilde{\alpha}_{MAX}$ to obtain the equivalents for statistical CSI:

- If S_S is a relay introduced on purpose by the network owner to enhance primary transmission, then $\alpha = 0$, since the primary transmitter receives all the benefits.
- If S_P sustains a *Constant Bit Rate* (CBR) service, such as a CBR video broadcast, a rate increase is useless for the primary transmitter. In this case, S_S could be rewarded with all the recycled resources ($\alpha = \alpha_{MAX}$).
- If both S_P and S_S are *Variable Bit Rate* (VBR) sources, they should share the released resources as fairly as possible. Usually, MAC protocols treat fairness in terms of channel access opportunities, ignoring the throughput limits of the users [16]. This is a sensible approach since, if throughputs were taken into account, giving priority to the best users would result in starvation. On the other hand, seeking equal throughput by assigning more resources to the worst users, would result in heavy resource under-utilization. The philosophy of throughput-agnostic fairness is also desirable in our design. Since the goal of cognitive radio is to gain spectrum resources, we think it would be fair to allocate half the spectrum gains to each user. Therefore, we suggest setting $\alpha = \alpha_{MAX}/2$, so that the primary transmitter is allocated its original capacity plus half the resource gains.

In the following sections we characterize α_{MAX} and $\tilde{\alpha}_{MAX}$ for different channel conditions. This provides insight into achievable gains, which are of interest regardless of how they are divided. In any case, given our assumptions, the less efficient S_P is, the more incentives S_S will have to cooperate.

IV. BENEFIT PROBABILITY

To determine whether or not cooperation might be beneficial, we will analyze the maximum M.I. that S_P achieves by cooperating when instantaneous CSI is available and S_S does not receive any resources ($\alpha = 0$). Cooperative diversity will be used if:

$$I_C > I_D \tag{9}$$

Next, we characterize statistically the probability of this event for a single-helper network under slow Rayleigh fading, in which cooperation is based on the DF protocol in [2] ($I_C \equiv I_{DF}$). As in [2], we use the exponential distribution of the squared absolute value of the fading coefficient $u_{ij} = |a_{ij}|^2$ ($|a| \sim Rayleigh(\sigma) \rightarrow u \sim Exp(\lambda), \lambda = \frac{1}{2\sigma^2}$). The event of interest is:

$$\begin{aligned} &I_{DF} > I_D \\ &\quad\Rightarrow \frac{1}{2}\log_2(1+\gamma\min(u_{sr}, u_{sd}+u_{rd})) > \log_2(1+\gamma u_{sd}) \end{aligned} \tag{10}$$

Or, equivalently:

$$\min(u_{sr}, u_{sd}+u_{rd}) > u_{sd}(2+\gamma u_{sd}) \tag{11}$$

The probability that it will take place, as proven in appendix A, is:

$$\begin{aligned} P[I_{DF} > I_D] &= \frac{\sqrt{\pi}\lambda_{sd}e^{\frac{\mu^2}{2}}Q(\mu)}{\sqrt{(\lambda_{rd}+\lambda_{sr})\gamma}}, \\ \mu &= \frac{(\lambda_{rd}+2\lambda_{sr}+\lambda_{sd})}{\sqrt{2(\lambda_{rd}+\lambda_{sr})\gamma}} \end{aligned} \tag{12}$$

Figures 3, 4 and 5 show examples of different behaviors. Each shows the benefit probability for a three-node network in which the fading parameters of two of the three channels (source to destination (SD), source to relay (SR) and relay to destination (RD)) are fixed, $\sigma_{ij} = 1$, and the remaining channel has a fading parameter of between 0.1 and 10. It can be seen that the probability of cooperation increases to 1 when the SD channel is worse than the other two (figure 3). In contrast, improvements in the other two channels do not necessarily yield resource gains (figures 4 and 5). This confirms the intuition that inefficient spectrum license holders should be encouraged to share their resources according to our model. Moreover, the influence of SNR without fading is appreciable, again confirming the intuition that low SNR

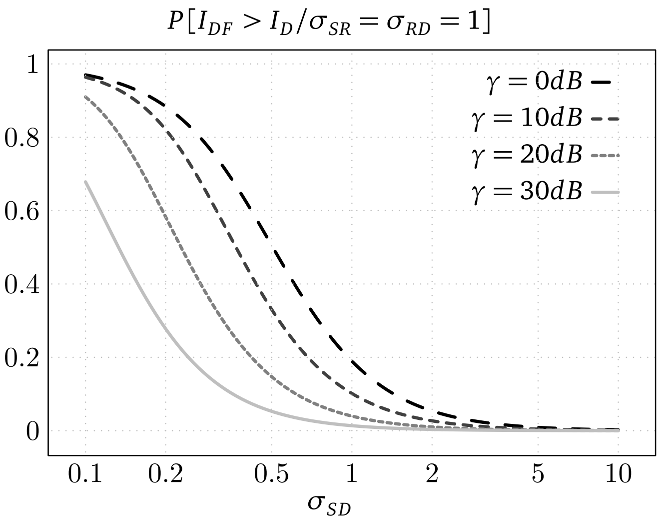

Figure 3. Probability of cooperation depending on SD channel statistics. Primary transmitters affected by poor conditions usually benefit greatly from cooperation.

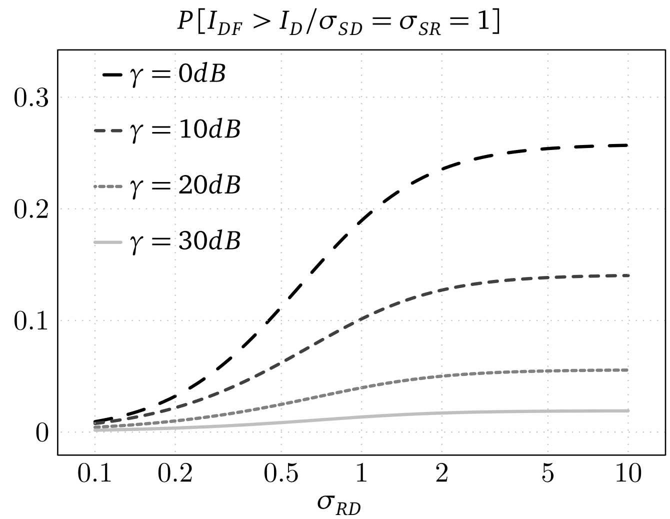

Figure 5. Probability of cooperation depending on RD channel statistics. If the relay-to-destination channel is good, there may be some resource gains, although this channel is less determinant than SD.

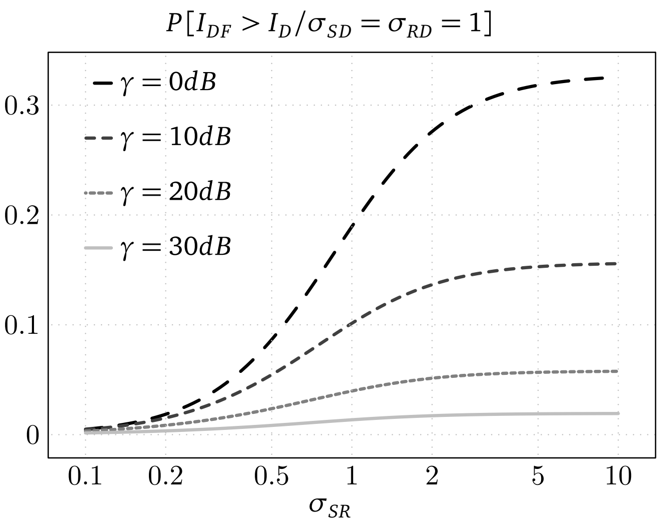

Figure 4. Probability of cooperation depending on SR channel statistics. If the source-to-relay channel is good, there may be some resource gains, although this channel is less determinant than SD.

systems tend to benefit more from cooperation. The fact that values of near one are achieved makes the potential benefits of the system very appealing.

V. Mutual Information Distribution

We have shown that there are channel scenarios in which cooperation improves efficiency. In this section and the following, we will consider M.I. to be a random variable dependent on channel state. We will now obtain the probability density function (p.d.f.) of the M.I. in the two transmission modes (D and DF). As in section IV, we employ a three-node topology in the analysis that follows. The M.I. of a given protocol X will be noted as I_X, its realizations as i_X, and the p.d.f.s as $f_{I_X}(i_X)$. For direct transmission the p.d.f. of the M.I. I_D is:

$$f_{I_D}(i_D) = \frac{\ln(2)}{\gamma} 2^{i_D} e^{-\lambda_{sd}\frac{2^{i_D}-1}{\gamma}} \tag{13}$$

For cooperative DF, to avoid the minimization in the logarithm, we use the partition property [17]. Let $\epsilon \equiv u_{sr} > u_{sd} + u_{rd}$ and let ϵ^c represent the complementary event. For compactness, let $P = P[\epsilon]$ and $P^c = 1 - P$.

$$f_{I_{DF}}(i_{DF}) = P f_{I_{DF}/\epsilon}(i_{DF}) + P^c f_{I_{DF}/\epsilon^c}(i_{DF}) \tag{14}$$

Expression (14) is completed by replacing the undefined components with (24) and then (25), as shown in appendix B.

Figures 6, 7 and 8 show a comparison of the M.I. for the direct and DF modes, covering three scenarios. In the first, all parameters σ_{ij} are set to 1 (corresponding to the so-called equilateral triangular network). The second scenario features a relay-in-the-middle disposition ($\sigma_{SD} = 0.5\sigma_{SR} = 0.5\sigma_{RD}$), and finally the third scenario represents a case with a lossy primary link with σ_{SD} ten times smaller than the parameters of the other links ($\sigma_{SD} = 0.1\sigma_{SR} = 0.1\sigma_{RD}$). All curves show channel realizations concentrated in "lobes" around the mean, which increases with SNR; they also show that the direct link lobes are wider than those of DF (whose lobes shift less to the right as SNR increases). This is due to the rate-halving nature of the DF protocol. In section VI we further elaborate on the mean of the M.I., which can be used as an estimation of the position of the lobes. The thinner lobe of DF is directly related to its outage behavior: this protocol exhibits a larger reduction of outage probability for a given reduction in rate.

From these plots, we can observe that DF M.I. is more stable against changes in the channel than that of direct transmission. DF does not produce the thick lower tails of direct transmission, but, on the other hand, it is more difficult for it to reach the peak M.I. values seen with direct transmission. The decision schema based on instantaneous CSI exploits the higher M.I. values of direct transmission when they are available, and it is not hampered by the lower ones because in those cases it opts for DF cooperation. The M.I. growth with SNR (lobe shifts to the right) is lower for DF than for direct transmission, because low-SNR systems tend to benefit more from cooperation. At a higher SNR it becomes more unlikely that the direct channel will be so faded that DF cooperation will be useful. Moreover, the relative positions of the two

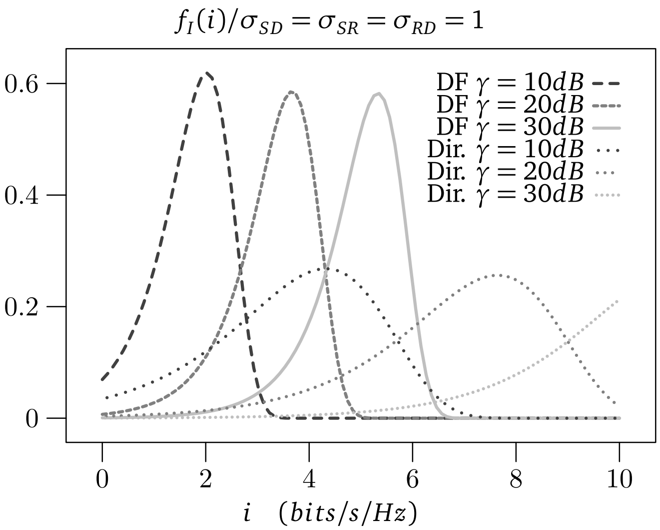

Figure 6. Mutual information p.d.f. of direct transmission and cooperative DF for the equilateral triangular layout. All DF p.d.f. lobes lie to the left of the corresponding direct transmission p.d.f.s. Decisions based on statistical CSI would select the direct mode and those based on instantaneous CSI would rarely opt for cooperative diversity when the direct channel is affected by deep fadings.

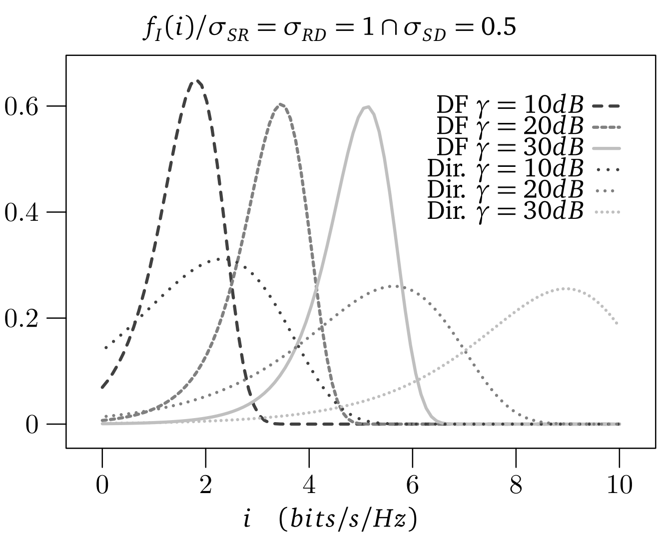

Figure 7. Mutual information p.d.f. of direct transmission and cooperative DF for the relay-in-the-middle layout. When the SNR is low, the p.d.f.s are highly overlapped. Thus, decisions based on instantaneous CSI would frequently switch between modes. DF lobes are still slightly shifted to the left compared to the respective direct transmission p.d.f.s. For higher SNRs the behavior is similar to that shown in figure 6.

functions (D and DF) as SNR changes depends on the channel parameter layout: the worse the SD channel is, the more likely cooperation is. Note that the DF lobes are even to the right of the direct transmission lobes in figure 8. All these statements confirm the intuitive interpretation of the previous section: the more inefficient the primary transmitter is, the more spectrum can be recycled for secondary transmission at typical SNR values, although the effect is more evident at low SNRs.

VI. Decision-Making Using Average Mutual Information

From the previous discussion, the benefit of cooperation decisions based on statistical CSI depends on the relative

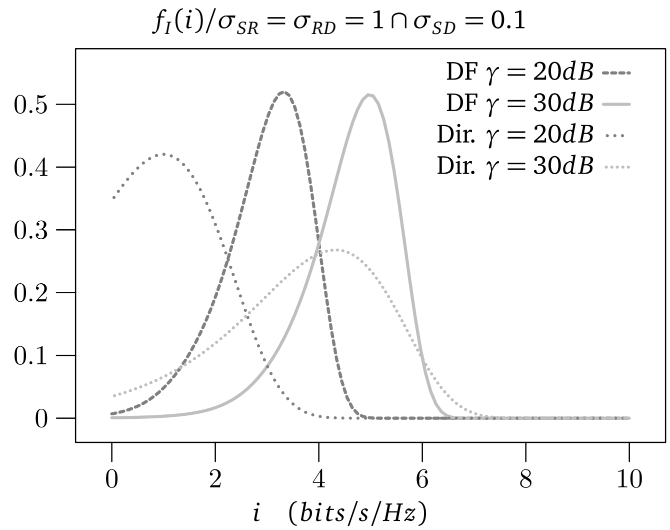

Figure 8. Mutual information p.d.f. of direct transmission and cooperative DF for the obstacles-in-primary transmitter-link layout. The two DF lobes are to the right of the corresponding direct mode p.d.f.s (we have omitted the $10dB$ case due to visibility issues because its peak value, 5, is much higher than in the other cases). Consequently, decisions based on statistical CSI will select the cooperative mode. Decisions based on instantaneous CSI will select DF in most cases, switching to direct mode on the rare occasions when only the source-to-destination channel is in good condition. If the SNR continues to increase, the system will return to the behavior shown in figures 6 and 7

positions of the lobes of the respective p.d.f.s. To implement a decisor with statistical CSI, the average M.I. can be computed from the statistics of the channel. This approach is interesting for rapidly changing channels or systems with complexity constraints.

To compute the average capacities we have developed the following expressions (see appendix C):

$$E[I_D] = \frac{1}{\ln(2)} e^{\frac{\lambda_{sd}}{\gamma}} \Gamma\left(0, \frac{\lambda_{sd}}{\gamma}\right) \tag{15}$$

$$E[(1-\alpha)I_{DF}] = (1-\alpha)\left[P^c E[I_{DF/\epsilon^c}] + P E[I_{DF/\epsilon}]\right] \tag{16}$$

The two terms in (16) correspond to expressions (29) and (32) if $\sigma_{sd} \neq \sigma_{rd}$ or to (30) and (33) if $\sigma_{sd} = \sigma_{rd}$, as shown in appendix C. It is important to recall that $1-\alpha$ is a constant factor in the derivation, and that it will be selected as $\alpha \in [0, \tilde{\alpha}_{MAX}]$. This interval is obtained using expression (8) together with (15) and (16).

Figure 9 shows the average M.I. curves for an SNR variation of between 0 dB and 30 dB and figure 10 shows the values of $\tilde{\alpha}_{MAX}$ resulting from these curves.

These figures show the three scenarios described in section V: equilateral-triangular, relay-in-the-middle and a lossy primary link. A fourth intermediate loss scenario has been added, with $\sigma_{SD} = 0.25$, to better illustrate the evolution of $\tilde{\alpha}_{MAX}$. The source-to-destination channel fading parameter is progressively increased to illustrate the performance loss: the average M.I. curves shift downwards as the channel worsens. The loss is more pronounced in the case of direct transmission.

Like figure 7, figure 10 shows that the direct mode is used for the equilateral triangular and relay-in-the-middle scenarios (thick-dotted and solid lines), since in these cases

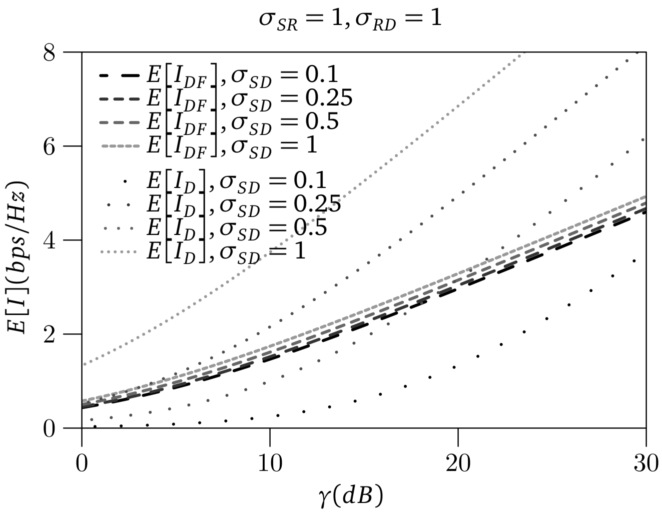

Figure 9. Average M.I. of cooperative DF and direct transmission versus SNR, for varying values of σ_{sd}. Cooperation is useful if average M.I. of DF exceeds that of direct transmission. I_D decreases substantially with SD channel degradation, whereas I_{DF} experiences comparatively smaller reductions. Cooperation therefore takes place in the case of permanent impairments, such as primary link obstacles, rather than in the case of fading.

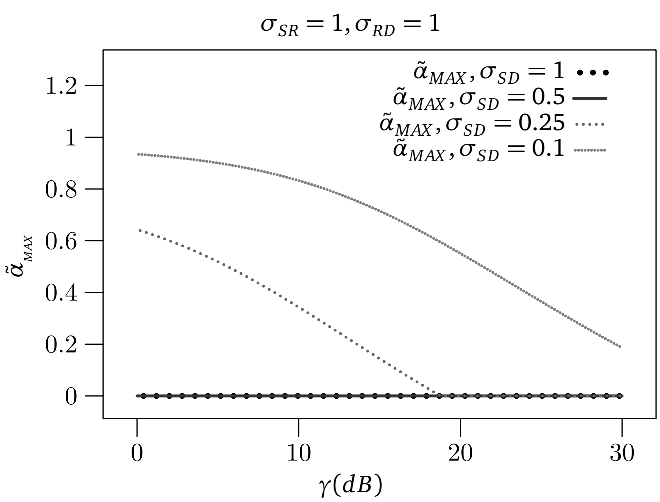

Figure 10. Proportion of resource gains for decisions based on statistical CSI ($\tilde{\alpha}_{MAX}$) versus SNR, for varying values of σ_{sd}. Cooperation is useful in all the regions where the proportion of resource gains is nonzero. For the SNR limit of $0dB$ in the $\sigma_{sd} = 0.1$ curve, over 90% of the resources are released. This value decreases considerably with increasing SNR. Resource gains decrease rapidly with improvements in the direct channel, leading to a complete lack of cooperation for $\sigma_{sd} = 0.5$

$\tilde{\alpha}_{MAX} = 0$. However, in the case with a worse source-to-destination channel ($\sigma_{SD} = 0.25$ and $\sigma_{SD} = 0.1$) resource gains appear and increase rapidly. Note that, for low SNRs, $\tilde{\alpha}_{MAX} = 0.64$ for $\sigma_{SD} = 0.25$, and it approaches 1 for $\sigma_{SD} = 0.1$ (93.4% resource gain). It is noteworthy that the resource gain seen in the case of statistical CSI is due to a primary link that is bad on average, rather than to temporary deep fading events. We can conclude that this is a low complexity approach for obtaining resources from primary transmitters affected by semi-permanent problems, such as obstacles or deficient antenna orientations, combined with random fading.

VII. Decision-Making Using Instantaneous Full CSI

Unlike the case described in the previous section, let us now suppose that S_P knows the channel realizations for transmission instantly. This schema can be used in systems that can afford extra complexity for a better exploitation of resources. In order to implement the decisor (3), inequality (11) yields a fairly simple criterion:

$$D_i = \arg\max(I_D, I_{DF}) = \begin{cases} DF & \text{if (11) is true} \\ D & \text{otherwise} \end{cases} \tag{17}$$

In this scenario the value of α_{MAX} is different for each channel realization. We treat this value as a random variable (A_{MAX}) and study its behavior ($f_{A_{MAX}}(\alpha_{MAX})$). We start with α_{th} because $f_{A_{th}}(\alpha_{th})$ is easier to derive.

$$f_{A_{th}}(\alpha_{th}) = \int_0^\infty i_c f_{I_D,I_C}((1-\alpha_{th})i_c, i_c) di_c \tag{18}$$

Since (6) may take negative values ($\alpha_{th} \in (-\infty, 1)$), the analysis of α_{MAX} is divided into two parts: $I_C < I_D$ (cooperation is not beneficial), which is equivalent to $P[A_{th} \leq 0]$, and $I_C > I_D$ featuring the distribution of positive values of α_{MAX}.

$$\begin{aligned} f_{A_{MAX}}(\alpha_{MAX}) =& P[A_{th} \leq 0]\delta(\alpha_{MAX}) \\ &+ P[A_{th} > 0] f_{A_{th}/A_{th}>0}(\alpha_{MAX}) \end{aligned} \tag{19}$$

This expression characterizes the resource gains achieved with cooperation. Unfortunately, for the DF protocol, there is no solution to (18)[1], so the study is carried out using numerical integration.

Figure 11 shows examples of $f_{A_{MAX}}(\alpha_{MAX})$. Once again, the observations in section IV are confirmed, in the sense that cooperation is particularly beneficial for bad primaries (note the concentration of high values of α_{MAX} for $\sigma_{SD} = 0.1$). The probability of cooperation ($P(\alpha_{MAX} > 0)$) and the average number of released resources ($E[\alpha_{MAX}]$) are computed in table I for the four scenarios in section VI: equilateral triangular, relay-in-the-middle and two levels of lossy primary link. For comparison, the last column of the table shows the benefits of statistical CSI. Note that for instantaneous CSI some benefits persist for high SNR values and good primaries. In fact, decisions based on instantaneous CSI can always generate nonzero resource gains but, as the primary link improves, these gains decrease to a point at which the channel estimation overhead may be unacceptable. Even though the development of a MAC protocol is beyond the aim of this paper, for a given protocol, the analytical results should be compared with an estimation of its overhead. If this results in negative net gains, it can be concluded that the protocol does not benefit from instantaneous CSI.

For example, in the relay-in-the-middle channel setting ($\sigma_{SD} = 0.5$), for transmissions at $\gamma = 10dB$, there are resource gains of $\sim 15\%$. Depending on the target MAC, if protocol overhead is below 15%, instantaneous CSI could be employed, whereas decisions based on statistical CSI would

[1]We obtained a sum of exponential integrals with polynomial exponents of order $(2 - \alpha_{th})$ without a closed-form solution.

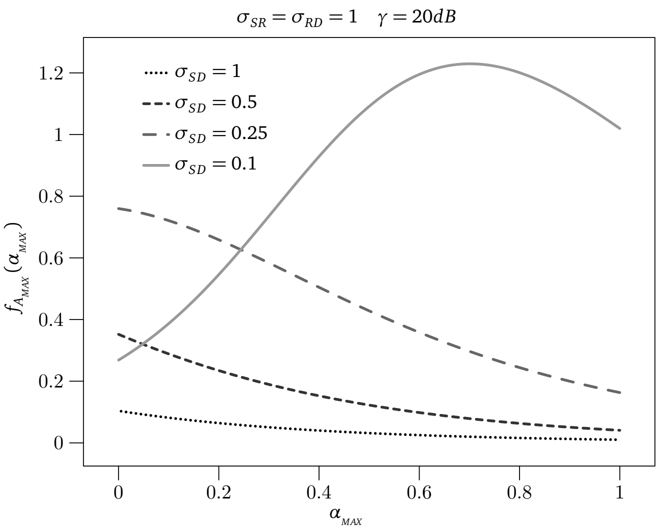

Figure 11. Distribution of positive α_{MAX} values in the four channel scenarios described in section VI. When the primary link is poor, resource gains tend to be large.

Table I
PROBABILITY OF COOPERATION AND AVERAGE RESOURCE GAINS WITH INSTANTANEOUS AND STATISTICAL CSI

σ_{sd}	γ	P_{coop}	$E[\alpha_{MAX}] \times 100\%$	$\tilde{\alpha}_{MAX} \times 100\%$
1	$0dB$	0.19	8.7%	0%
	$10dB$	0.10	4.0%	0%
	$20dB$	0.04	1.3%	0%
	$30dB$	0.01	0.3%	0%
0.5	$0dB$	0.51	27.0%	0%
	$10dB$	0.33	14.2%	0%
	$20dB$	0.15	4.9%	0%
	$30dB$	0.05	1.4%	0%
0.25	$0dB$	0.82	57.5%	64.3%
	$10dB$	0.72	39.2%	34.3%
	$20dB$	0.45	17.7%	0%
	$30dB$	0.19	5.2%	0%
0.1	$0dB$	0.97	93.1%	93.4%
	$10dB$	0.96	79.8%	83.2%
	$20dB$	0.91	53.5%	55.1%
	$30dB$	0.68	24.0%	18.6%

discourage cooperation in such a case. In the extreme scenario with $\sigma_{SD} = 0.1, \gamma = 0dB$, where decisions based on statistical CSI are effective, instantaneous CSI-assisted decisions also achieve resource gains above 90%. We conclude that the approach with instantaneous CSI is suitable in more cases than the statistical CSI approach: it provides similar gains from primary transmitters with permanent problems, but it can also extract resources from the temporary degradation of good primary channels. This behavior is akin to the philosophy of cooperative diversity, which relies on varying channel conditions such as fading.

VIII. CONCLUSIONS

The aim of cooperative diversity is to improve the efficiency of radio resource utilization without the need for bulky antennas in portable devices. As mobile communications continuously demand new features, opening peer cooperation to spectrum domains with heterogeneous priorities will boost cooperative diversity as an enabling technology for spectrum leasing in cognitive radio.

Previous models of spectrum leasing have been based on game theory. We propose a simplified model characterized by resource gains achieved by cooperation. The model represents a scenario where secondary transmitters naturally tend to help inefficient spectrum owners (i.e. primary transmitters) to mutual benefit. Because spectrum owners retain control of the whole process, leasing is not obscured by complexity of game strategies.

We have identified literature on physical and MAC layers for cooperative diversity that can be easily adapted to spectrum leasing according to the requirements of our model.

Two decision-making schemes are considered: a statistical CSI scheme and an instantaneous CSI scheme.We have provided an analytical characterization of cognitive leasing in cooperative diversity, and discussed the conditions in which the M.I. of a cooperative channel exceeds that of the direct channel for the two schemes.

We have also formulated the statistical distribution of M.I. and used this to derive the average M.I. gain of the statistical CSI schema and the p.d.f. of resource gains for the instantaneous CSI schema. Even though there are no resource gains in the first schema when the primary channel is good, they increase considerably in the opposite case. The second schema produces gains in all setups, and these increase progressively as primary channels get worse.

The resource gains resulting from the statistical CSI schema are mostly related to persistent impairments, such as those due to obstacles or poor installations, whereas those resulting from the instantaneous CSI schema depend both on permanent impairments and temporary fading. Both schemes provide large resource gains when the primary channel is bad. For many applications, statistical CSI would be sufficient because it would require lower signaling overhead than instantaneous CSI, and yet offer similar gains. However, instantaneous CSI offers moderate gains for many more channel conditions, and thus its utilization by some MAC protocols should not be ruled out. We conclude that cooperative diversity has excellent potential for use in spectrum reutilization in future cognitive wireless networks.

APPENDIX A
SOLUTION TO THE BENEFIT PROBABILITY INTEGRAL

$$
\begin{aligned}
P[I_{coop} > I_D] &= P[\min(u_{sr}, u_{rd} + u_{sd}) > u_{sd}(2 + \gamma u_{sd})] \\
&= \int_{-\infty}^{\infty} P[u_{rd} > (x + \gamma x^2)] \\
&\quad \times P[u_{sr} > (2x + \gamma x^2)] f_{u_{sd}}(x) dx \\
&= \lambda_{sd} \int_{0}^{\infty} e^{-(\lambda_{rd} + 2\lambda_{sr} + \lambda_{sd})x + (\lambda_{rd} + \lambda_{sr})\gamma x^2)} dx
\end{aligned} \tag{20}
$$

The following integration pattern is used:

$$
\int_0^\infty e^{-(C_1 x + C_2 x^2)} dx = \frac{\sqrt{\pi} e^{\frac{\mu^2}{2}} Q(\mu)}{\sqrt{C_2}}, \quad \mu = \frac{C_1}{\sqrt{(2C_2)}} \tag{21}
$$

where $Q(x)=\int_x^\infty \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}dx$ is the upper tail integral of the normal distribution. Replacing $C_1=\lambda_{rd}+2\lambda_{sr}+\lambda_{sd}$ and $C_2=(\lambda_{rd}+\lambda_{sr})\gamma$:

$$P[I_{coop}>I_D]=\frac{\sqrt{\pi}\lambda_{sd}e^{\frac{\mu^2}{2}}Q(\mu)}{\sqrt{(\lambda_{rd}+\lambda_{sr})\gamma}} \qquad \mu=\frac{(\lambda_{rd}+2\lambda_{sr}+\lambda_{sd})}{\sqrt{2(\lambda_{rd}+\lambda_{sr})\gamma}} \tag{22}$$

APPENDIX B
MUTUAL INFORMATION PROBABILITY DENSITY FUNCTIONS

For direct transmission, we use:

$$f_{I_D}(i_D)=\frac{\partial P(I_D<i_D)}{\partial i_D}=\frac{\ln(2)}{\gamma}2^{i_D}f_{u_{sd}}(\frac{2^{i_D}-1}{\gamma}) \tag{23}$$

For DF, we start with (14) from section V, and replace:

$$f_{I_{DF/\epsilon^c}}(i)=f_{u_{sr}/\epsilon^c}\left(\frac{2^{2i}-1}{\gamma}\right)\frac{\partial\frac{2^{2i}-1}{\gamma}}{\partial i} \qquad f_{I_{DF/\epsilon}}(i)=f_{u_{sd}+u_{rd}/\epsilon}\left(\frac{2^{2i}-1}{\gamma}\right)\frac{\partial\frac{2^{2i}-1}{\gamma}}{\partial i} \tag{24}$$

It is not necessary to obtain the partition probabilities of (14) because the conditional p.d.f.s obtained using the Bayes theorem [17] will cancel these:

$$f_{u_{sd}+u_{rd}/\epsilon}(x)=\frac{1-F_{u_{sr}}(x)}{P}f_{u_{sd}+u_{rd}}(x) \qquad f_{u_{sr}/\epsilon^c}(x)=\frac{1-F_{u_{sd}+u_{rd}}(x)}{P^c}f_{u_{sr}}(x) \tag{25}$$

APPENDIX C
AVERAGE MUTUAL INFORMATION

The average M.I. for the direct transmission is:

$$E[I_D]=\int_{-\infty}^{\infty}\log_2(1+\gamma x)f_{u_{sd}}(x)dx =\frac{1}{\ln(2)}\int_0^\infty \ln(1+\gamma x)\lambda_{sd}e^{-\lambda_{sd}x}dx \tag{26}$$

Using integration by parts it can be compacted to an incomplete gamma function:

$$E[I_D]=\frac{1}{\ln(2)}e^{\frac{\lambda_{sd}}{\gamma}}\Gamma\left(0,\frac{\lambda_{sd}}{\gamma}\right) \tag{27}$$

For the DF protocol, the two parts of the partitioned expression (16) in section VI are:

A. First Term

The first term of the M.I. is:

$$P^cE[I_{DF/\epsilon^c}]=P^c\int_0^\infty\frac{1}{2}\log_2(1+\gamma x)f_{sr/\epsilon^c}(x)dx \tag{28}$$

Solving it for $\lambda_{sd}\neq\lambda_{rd}$ using integration by parts, the expression can be written as:

$$\frac{\lambda_{sr}}{2\ln(2)(\lambda_{rd}-\lambda_{sd})}\times\left[\frac{\lambda_{rd}G\left(\frac{(\lambda_{sr}+\lambda_{sd})}{\gamma}\right)}{\lambda_{sr}+\lambda_{sd}}-\frac{\lambda_{sd}G\left(\frac{(\lambda_{sr}+\lambda_{rd})}{\gamma}\right)}{\lambda_{sr}+\lambda_{rd}}\right] \tag{29}$$

with $G(x)=e^x\Gamma(0,x)$.

On the other hand, for $\lambda_{sd}=\lambda_{rd}=\lambda$

$$\frac{\lambda_{sr}}{2\ln(2)(\lambda_{sr}+\lambda)}\times\left[\left(\frac{\lambda}{\lambda+\lambda_{sr}}+1-\frac{\lambda}{\gamma}\right)G\left(\frac{(\lambda_{sr}+\lambda)}{\gamma}\right)+\frac{\lambda}{(\lambda_{sr}+\lambda)}\right] \tag{30}$$

B. Second Term

The second term of the M.I. is:

$$PE[I_{DF/\epsilon}]=P\int_{-\infty}^{\infty}\frac{1}{2}\log_2(1+\gamma x)f_{u_{rd}+u_{sd}/\epsilon}(x)dx =\frac{1}{2\ln(2)}\int_{-\infty}^{\infty}\ln(1+\gamma x)[1-F_{u_{sr}}(x)]f_{u_{rd}+u_{sd}}(x)dx \tag{31}$$

When $\lambda_{sd}\neq\lambda_{rd}$, the integral is very similar to the first term:

$$\frac{\lambda_{sd}\lambda_{rd}}{2\ln(2)(\lambda_{rd}-\lambda_{sd})}\times\left[\frac{G\left(\frac{(\lambda_{sr}+\lambda_{sd})}{\gamma}\right)}{\lambda_{sr}+\lambda_{sd}}-\frac{G\left(\frac{(\lambda_{sr}+\lambda_{rd})}{\gamma}\right)}{\lambda_{sr}+\lambda_{rd}}\right] \tag{32}$$

Otherwise, for $\lambda_{sd}=\lambda_{rd}=\lambda$, with similar integration:

$$\frac{\lambda}{2\ln(2)(\lambda+\lambda_{sr})}\times\left[\left(\frac{\lambda}{(\lambda+\lambda_{sr})}-\frac{\lambda}{\gamma}\right)G\left(\frac{(\lambda_{sr}+\lambda)}{\gamma}\right)+\frac{\lambda}{(\lambda+\lambda_{sr})}\right] \tag{33}$$

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